GCE AS/A level

0981/01

MATHEMATICS – M2
Mechanics

A.M. THURSDAY, 6 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. An object of mass 8 kg slides in a straight line from point A to point B on a rough horizontal floor. At A, the speed of the object is $7\,\text{ms}^{-1}$. It is brought to rest at B by a constant frictional force between the object and the floor. The distance AB is 15 m.

(a) Calculate the loss in kinetic energy. [2]

(b) Determine the coefficient of friction between the object and the floor. [4]

2. A particle P, of mass 2 kg, is moving so that at time t s its velocity $\mathbf{v}\,\text{ms}^{-1}$ is given by $\mathbf{v} = (13t - 3)\mathbf{i} + (2 + 3t^2)\mathbf{j}$. At time $t = 0$ s, the position vector of the particle is $(2\mathbf{i} + 7\mathbf{j})$ m.

(a) Find the position vector $\mathbf{r}$ of P at time t s. [5]

(b) Determine the acceleration $\mathbf{a}$ of P at time t s. [2]

(c) Calculate the values of t when the velocity of P is perpendicular to the vector $\mathbf{i} - 2\mathbf{j}$. [5]

3. A person throws a ball from a point A to hit a vertical pole, which is placed at a horizontal distance of 9 m from A. The point A is 1 m above the horizontal ground. The ball is projected with initial speed $15\,\text{ms}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$.

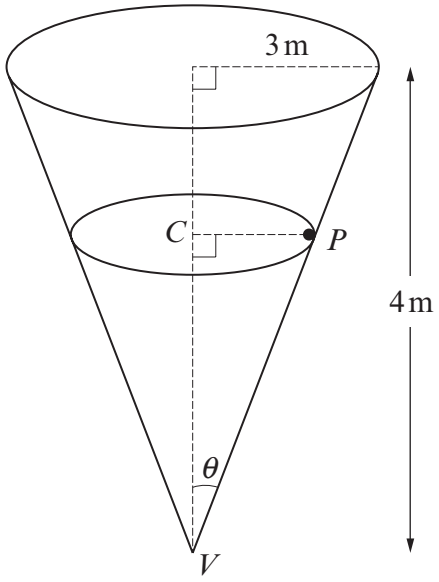
(a) Given that the ball hits the pole at a point B,

(i) find the time taken for the ball to reach B,

(ii) determine the height of B above the ground. [7]

(b) Given that the ball misses the pole and hits the ground, calculate the speed with which it hits the ground. [5]

4. The diagram shows a hollow cone, of base radius 3 m and height 4 m, which is fixed with its axis vertical and vertex V downwards. A particle P, of mass M kg, moves in the horizontal circle with centre C on the smooth inner surface of the cone with constant speed $\sqrt{\frac{8g}{3}}$ ms^{-1}, where g ms^{-2} is the acceleration due to gravity.



(a) Show that the normal reaction of the surface of the cone on the particle is $\frac{5Mg}{3}$ N. [4]

(b) Calculate the length of CP and hence determine the height of C above V. [5]

5. A particle moves along a straight horizontal line. Its velocity v ms^{-1} at time t s, is given by

$$v = 2t(t - 6).$$

(a) Find the set of values of t for which the velocity of the particle is negative. [2]

(b) Find the total distance travelled by the particle in the interval $0 \leqslant t \leqslant 9$. [5]

6. A car of mass 1500 kg is towing a trailer of mass 500 kg by means of a rigid tow bar up a slope inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{14}$.

The resistance to motion acting on the car is 170 N and that acting on the trailer is 30 N. The car's engine is working at a constant rate of 60 kW. When the car and the trailer are moving at a speed of 20 ms^{-1},

(a) calculate the tractive force acting on the car, [2]

(b) show that the acceleration of the car and the trailer is 0·7 ms^{-2}, [4]

(c) determine the tension in the tow bar. [4]

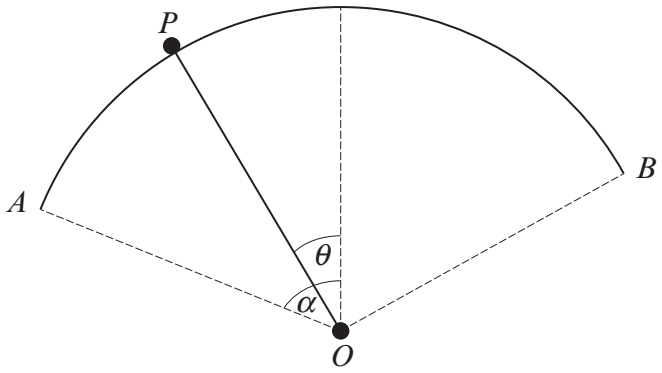
TURN OVER

7. The end A of a light elastic string AB, of natural length 1·2 m and modulus of elasticity 360 N, is fixed. A particle P, of mass 2 kg, is attached to the end B. Initially, P is held at rest at a point which is 0·7 m vertically below A. It is then released and allowed to fall.

(a) Find the greatest extension of the string in the subsequent motion. Give your answer correct to 2 decimal places. [7]

(b) Calculate the velocity of the particle when it is 1·2 m below A. [4]

8. The diagram shows a particle of mass 3 kg at a point P on the smooth outer surface AB of a sphere centre O and radius 4 m. The points O, A, P and B are in the same vertical plane.



Initially, the particle is held at rest at the point A, where OA makes an angle α with the upwards vertical and $\cos \alpha = 0{\cdot}8$. The particle is then projected with velocity 5 ms^{-1} in a direction which is perpendicular to OA, so that the particle moves along the arc AB. When the particle is at P, OP makes an angle θ with the upwards vertical.

(a) Find, in terms of θ, the speed of the particle at P. [4]

(b) Determine, in terms of θ, the reaction between the particle and the sphere at P. [4]

